NELSON MANDELA UNIVERSITY

Faculty of Engineering, the Built Environment & IT
Technology for tomorrow

Department of Mechatronics
Advanced Manufacturing Systems IV
EAMV401

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ASSIGNMENT 2

DYNAMICS & KINEMATICS FOR 6-DOF ROBOT

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Assignment Declaration

Module Name: Advanced Manufacturing Systems IV

Module Code: EAMV401

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Date: May 2024

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Nomenclature

DOF	Degrees of Freedom
DH	Denavit-Hartenberg
EE	End-Effector
KRL	KUKA Robot Language

Introduction

In the field of robotics, understanding the dynamics and kinematics of industrial robots is essential for optimising their performance. This report aims to demonstrate an understanding of the kinematics and dynamics associated with a 6-DOF (Degrees of Freedom) robotic manipulator, specifically the KUKA KR30HA robot. The focus is on deriving an appropriate motion solution industrial robots. The purpose of this assignment is to show an understanding of manipulator kinematics and dynamics by successfully arriving at an appropriate motion solution for a commercial Industrial robot.

Problem Description

The primary objective of this assignment is to perform a detailed analysis of the KUKA KR30HA robot in Figure 1, to achieve accurate motion control from an initial position to specified target points and back. This involves determining the robot's forward and inverse kinematics and deriving the equations of motion for dynamic analysis. A KUKA Robot Language (KRL) program that can execute a defined motion path efficiently is required.

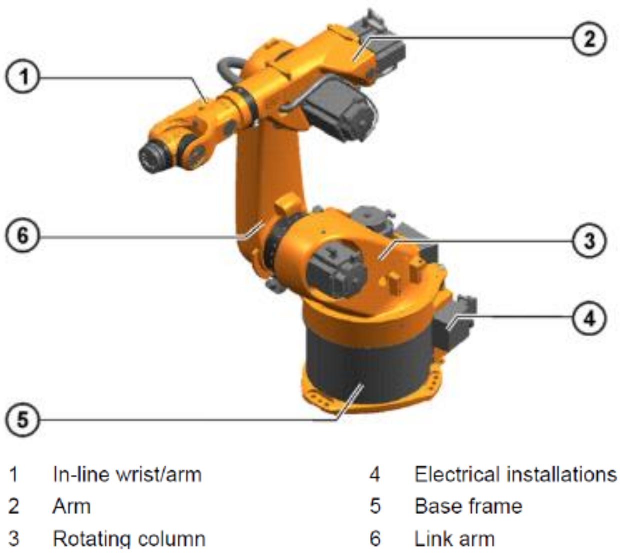

Figure 1: KR30HA robot with component labels

The figure above shows the robots physical structure, identifying its key components. To determine the coordinate location of the robot, the mastering positions will need to be used together with label 5, the base frame.

Methodology

This section outlines the steps taken to solve each of the three problems mentioned in the previous section. Furthermore, the kinematic and dynamic analysis conducted to obtain an appropriate motion solution for the KUKA KR30HA robot is described with the use of equations. A set of coordinates, with respect to the Global/World coordinate system, for two points in space, illustrated in Table 2, were given.

Table 1: Motion point coordinates

Point 1 [mm]			Point 2 [mm]		
X	Y	Z	X	Y	Z
91	-1139	1392	91	-1117	1447

The coordinates of two points in space and will prove significant in calculating the inverse kinematics of the system to find the joint angles corresponding to the values in Table 1.

Forward Kinematics

The initial task was to determine the forward kinematics of the system to determine the position and orientation of the end-effector, given the joint parameters of the robot. To do this, the following three steps were taken:

1. Find the Denavit-Hartenberg (DH) Parameters of the robot.
    a. The DH parameters were extracted from the KUKA KR30HA technical manual, including the lengths (a_i), link offsets (d_i), link twists (α_i), and joint angles (θ_i).
2. Determine the transformation matrices.
    a. The transformation matrices were obtained using the DH parameters and the general equation below.

$$A_i = \begin{bmatrix} C\theta_i & -S\theta_i C\alpha_i & S\theta_i S\alpha_i & \alpha_i C\theta_i \\ S\theta_i & C\theta_i C\alpha_i & -C\theta_i S\alpha_i & \alpha_i S\theta_i \\ 0 & S\alpha_i & C\alpha_i & d_i \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{1}$$

3. Calculate the composite transformation matrix.
    a. The overall transformation matrix was calculated by multiplying the individual transformation matrices from the base frame to the end-effector frame as shown in the equation below.

$$\mathrm{T}_{\mathrm{TCP}}^{0} = T_1^0 T_2^1 T_3^2 \ldots T_{TCP}^5 = \begin{bmatrix} l_x & m_x & n_x & p_x \\ l_y & m_y & n_y & p_y \\ l_z & m_z & n_z & p_z \\ 0 & 0 & 0 & 1 \end{bmatrix} \tag{2}$$

Inverse Kinematics

The second step was to determine the inverse Kinematics of the system to determine the joint angles of the robot that achieve the two point locations in Table 2. This was achieved by following the steps below:

1. Define the target positions for the robot.
    a. Identify the coordinates of the home position, point 1 and point 2 in the global coordinate system. These positions included the $X, Y, and\ Z$ coordinates and the orientation (Yaw, Pitch and Roll).

$$\text{Home Position: } (X_0, Y_0, Z_0)$$

$$\text{Point 1: } (X_1, Y_1, Z_1)$$

$$\text{Point 2: } (X_2, Y_2, Z_2)$$

2. Solve inverse kinematics.
    a. Analytical methods were used to solve the inverse kinematics equations, which involved solving equations directly for the first three joints. The process involved:
        i. Applying geometric and algebraic methods to find the angles based on the provided dimensions and target positions.
        ii. Determining the joint angles $\theta_1, \theta_2, \text{and}\ \theta_3$ that achieve those end positions.

Equations of Motion

Lastly, the equations of motion for the first two joints and links were obtained, considering both kinematic and dynamic aspects of the robot.

1. Kinematic Analysis
    a. The velocity and acceleration of the first two joints and links were calculated using the joint parameters (angles) and their derivatives.
2. Dynamic analysis using Euler-Lagrange Formulation.
    a. The Lagrangian equation can be seen in Equation 3, where K = total kinetic energy and P = the total potential energy of the first two joints and links.

$$L = K - P \tag{3}$$

    b. The translational and rotational kinetic energy terms for each link were calculated using Equations 4 and 5 respectively. The total kinetic energy was then calculated using Equation 6.

$$K_{i:TRANSLATIONL} = \frac{1}{2} m_i v_i^2 \tag{4}$$

$$K_{i:ROTATIONAL} = \frac{1}{2} I_i \omega_i^2 \tag{5}$$

$$K_{TOTAL} = \frac{1}{2}\sum_{i=1}^{2} K_{i:TRANSLATIONL} + \frac{1}{2}\sum_{i=1}^{2} K_{i:ROTATIONAL} \tag{6}$$

where m_i - mass of the link
V_i – linear velocity
$I_i = m_i r^2$ – moment of inertia
r = distance to point of rotation

c. The gravitational potential energy and the total potential energy of the two was calculated.

$$P_{i:GRAVITATIONAL} = m_i g h_i \tag{7}$$

$$P_{TOTAL} = \sum_{i=1}^{2} P_{i:GRAVITATIONAL} \tag{8}$$

where g – gravitational acceleration
h_i – height to the centre of mass of link i

d. The Lagrange equation, Equation 9, was used to solve the solution.

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q_i}}\right) - \left(\frac{\partial L}{\partial q_i}\right) = \tau_i \tag{9}$$

where q_i – joint angle
τ_i – torque at the joints

3. Programming
    a. A requirement of this assignment was to write a KUKA Robot Language (KRL) program that performs the motion from the mastering position to Point 1, then to Point 2, and back to the mastering position.
    b. This involved initialising the robot, declaring positions of the points and writing several move commands.

This section outlines the structured approach taken to perform the kinematic and dynamic analysis of the KUKA KR30HA robot. Finding the forward and inverse kinematics, and equations of motion, assisted in finding an appropriate solution for a commercial robot. The results of applying the above calculations are explained in the following sections of the report.

Kinematic Calculations

A schematic of the robot under investigation is illustrated in Figure 2, indicating the directions of joint rotations.

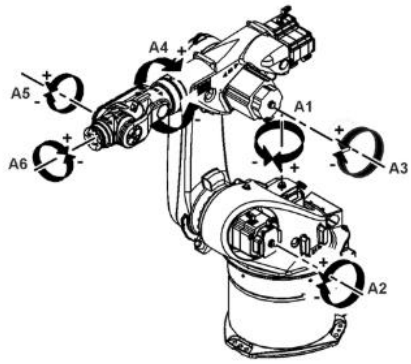

Figure 2: Robot axis rotations

The rotational axes in Figure 2 and the working envelope in Figures 4 and 5 of the KUKA robot will prove significant in the calculations throughout this section.

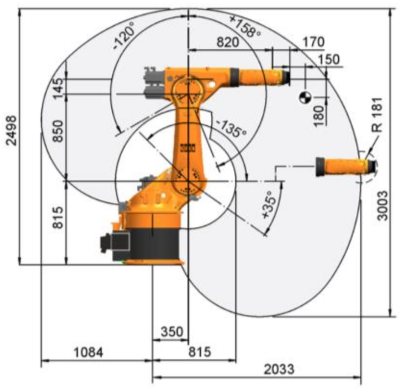

Figure 3: Working envelope (side view)

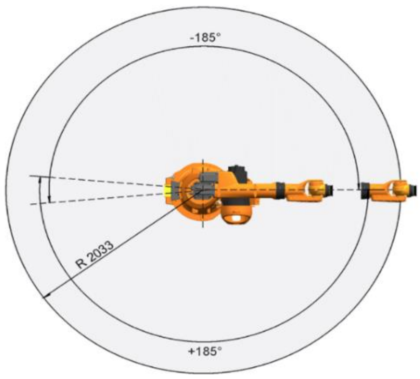

Figure 4: Working envelope (top view)

The first step in solving the industrial robot solution involved forward kinematics to find the position of the EE with respect to the robot base frame.

Forward Kinematics

Joint 1 was assumed to be located at the base of the robot and using Figure 2 from the previous section, the x and y coordinate frames were assigned according to the Denavit-Hartenberg Convention and illustrated in Figure 6. The following important rules were followed to obtain the frames:

1. z-axis is in the direction of rotation (right-hand rule).
2. Axis $x_i \perp$ to axis z_{i-1}.
3. $\mathrm{x_i}$ - axis must intersect the $\mathrm{z_{i-1}}$ - axis.

4. x-axis in the direction of the common normal, which is the shortest line mutually perpendicular between any two skew lines.

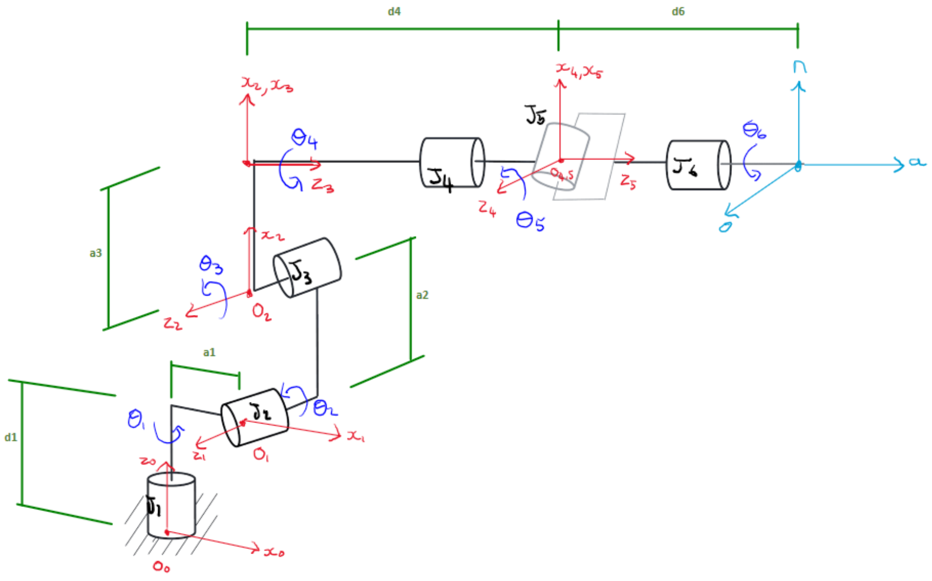

Figure 5: DH Convention robot frames

This diagram played an essential role in determining the DH parameters in Table 2.

Table 2: DH Convention Parameters

Link no.	$\boldsymbol{\theta_i}$	$\boldsymbol{d_i}$	$\boldsymbol{a_i}$	$\boldsymbol{\alpha_i}$
0 - 1	θ_1^*	d_1	a_1	$90°$
1 - 2	$\theta_2^* + 90°$	0	a_2	$0°$
2 - 3	θ_3^*	0	a_3	$90°$
3 - 4	θ_4^*	d_4	0	$-90°$
4 - 5	θ_5^*	0	0	$90°$
5 - TCP	θ_6^*	d_6	0	$0°$

Table 3 was used to find the homogeneous transformation matrices for each of the links.

$$T_1^0 = Trans(x, a_1) \cdot Trans(z, d_1) \cdot Rot\left(x, \frac{\pi}{2}\right) \cdot Rot(z, \theta_1^*)$$

$$T_2^1 = Trans(x, a_2) \cdot Rot(z, \theta_2^* + 90°)$$

$$T_3^2 = Trans(x, a_3) \cdot Rot\left(x, \frac{\pi}{2}\right) \cdot Rot(z, \theta_3^*)$$

$$T_4^3 = Trans(z, d_4) \cdot Rot\left(x, -\frac{\pi}{2}\right) \cdot Rot(z, \theta_4^*)$$

$$T_5^4 = Rot\left(x, \frac{\pi}{2}\right) \cdot Rot(z, \theta_5^*)$$

$$T_{TCP}^5 = Trans(z, d_6) \cdot Rot(z, \theta_6^*)$$

Using Equation 1 to find A_1 to A_6 yields the following homogeneous transformations:

$$T_1^0 = \begin{bmatrix} C\theta_1 & 0 & -S\theta_1 & a_1C\theta_1 \\ S\theta_1 & 0 & C\theta_1 & a_1S\theta_1 \\ 0 & -1 & 0 & d_1 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2^1 = \begin{bmatrix} S\theta_2 & -C\theta_2 & 0 & a_2S\theta_2 \\ C\theta_2 & S\theta_2 & 0 & a_2C\theta_2 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_3^2 = \begin{bmatrix} C\theta_3 & 0 & -S\theta_3 & a_3C\theta_3 \\ S\theta_3 & 0 & C\theta_3 & a_3S\theta_3 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} C\theta_4 & 0 & S\theta_4 & 0 \\ S\theta_4 & 0 & -C\theta_4 & 0 \\ 0 & 1 & 0 & d_4 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_5^4 = \begin{bmatrix} C\theta_5 & 0 & -S\theta_5 & 0 \\ S\theta_5 & 0 & C\theta_5 & 0 \\ 0 & -1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_{TCP}^5 = \begin{bmatrix} -C\theta_6 & S\theta_6 & 0 & 0 \\ -S\theta_6 & -C\theta_6 & 0 & 0 \\ 0 & 0 & 1 & d_6 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

The distances and angles in Table 4 were found using the given robot's working envelope in Figures 4 and 5 and the mastering positions (A0-A6) of the robot in Appendix A.

Table 3: DH Distances

Distance	Value
d_1	815 mm
a_1	350 mm
a_2	850 mm
a_3	145 mm
d_4	820 mm
d_6	170 mm

By making use of Equation 1 and the mastering positions of the robot, the following HT matrices were derived.

$$T_1^0 = \begin{bmatrix} 1 & 0 & 0 & 350 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 815 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_2^1 = \begin{bmatrix} 1 & 0 & 0 & 850 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_3^2 = \begin{bmatrix} 0 & 0 & 1 & 0 \\ 1 & 0 & 0 & 145 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^3 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & -1 & 0 & 820 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_5^4 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_{TCP}^5 = \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 170 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Using Equation 2, the overall HT matrix can be found.

$$T_2^0 = T_1^0T_2^1 = \begin{bmatrix} 1 & 0 & 0 & 1200 \\ 0 & 0 & -1 & 0 \\ 0 & 1 & 0 & 815 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_3^0 = T_2^0T_3^2 = \begin{bmatrix} 0 & 0 & 1 & 1200 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 960 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

$$T_4^0 = T_3^0T_4^3 = \begin{bmatrix} 0 & -1 & 0 & 2020 \\ 0 & 0 & -1 & 0 \\ 1 & 0 & 0 & 960 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad T_5^0 = T_4^0T_5^4 = \begin{bmatrix} 0 & 0 & 1 & 2020 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 960 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Finally, the overall transformation matrix for the KR30HA robot is given as:

$$T_{TCP}^0 = T_5^0T_6^5 = \begin{bmatrix} 0 & 0 & 1 & 2190 \\ 0 & -1 & 0 & 0 \\ 1 & 0 & 0 & 960 \\ 0 & 0 & 0 & 1 \end{bmatrix}$$

Inverse Kinematics

Inverse kinematics involved determining the joint angles, θ_1, θ_1 and θ_1 given a desired position and orientation of the third link. This requires solving for the joint variables that achieve a specific pose.

For the inverse kinematics, the elbow up analogy was used to find the angles. The figure below illustrates the elbow up configuration of the robot for the first three links.

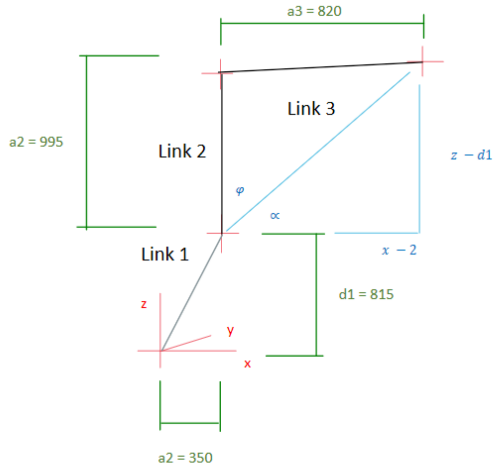

Figure 6: KUKA robot elbow up configuration

An assumption was made for the length of link two, this being 820 mm. due to this, the length of link two was calculated as 995 mm which was a result of adding a_2 and a_3 from Table 4. The robot is required to move from mastering position to Point 1, to Point 2, and then back to mastering position. Points 1 and 2 can be found in Table 2, whereas the home position for the end of the third link was determined geometrically using Figure 3 of the robots working envelope.

$$\text{Point } 1(91, -1139, 1392)$$

$$\text{Point } 2(91, -1117, 1447)$$

$$\text{Home}(1310, 0, 1810)$$

These coordinates were used to determine the inverse kinematics for the first three joints. The inverse kinematics calculations were performed to the end of link 3 and link 4 onwards was not included. Calculating the joint angles $\theta_1, \theta_2,$ and θ_3 required the following formulae that were applied by substituting in the values from Table 5.

$$D = \frac{(x_c - d_2)^2 + y_c^2 + (z_c - d_1)^2 - a_2^2 - a_3^2}{2a_2a_3}$$

$$\alpha = \tan^{-1}\left(\frac{z_c - d_1}{\sqrt{(x_c - d_2)^2 + y_c^2}}\right)$$

$$\varphi = \sin^{-1}\left(\frac{a_3 \sin\theta_3}{\sqrt{(x_c - d_2)^2 + y_c^2 + (z_c - d_1)^2}}\right)$$

Table 4: Inverse kinematics parameters

z-offset d1	815
x-offset d2	350
Link 2 a2	995
Link 3 a3	820

Point 1 [mm]			Point 2 [mm]			Home [mm]		
X	Y	Z	X	Y	Z	X	Y	Z
91	-1139	1392	91	-1117	1447	1170	0	1810

1. Calculations for Point 1:

$$\Theta_1 = \tan^{-1}\left(\frac{y_c}{x_c}\right) = -102.81°$$
$$D = 0.021391$$
$$\Theta_3 = \tan^{-1}\left(\frac{\sqrt{1-D^2}}{D}\right) = -88.78°$$
$$\Theta_2 = \alpha \mp \varphi$$
$$\alpha = 26.29°$$
$$\varphi = -39.0°$$
$$\Theta_2 = \alpha \mp \varphi = 26.29° \pm (-39.0°) = -12.71° \; or \; 65.28°$$

2. Calculations for Point 2:

$$\Theta_1 = \tan^{-1}\left(\frac{y_c}{x_c}\right) = -103.01°$$
$$D = 0.03173$$
$$\Theta_3 = \tan^{-1}\left(\frac{\sqrt{1-D^2}}{D}\right) = -88.18°$$
$$\alpha = 28.86°$$
$$\varphi = -38.75°$$
$$\Theta_2 = \alpha \mp \varphi = 28.86° \pm (-38.75°) = -9.89° \; or \; 67.62°$$

3. Calculations for Home position:

$$\Theta_1 = \tan^{-1}\left(\frac{y_c}{x_c}\right) = 0°$$
$$\Theta_3 = 90°$$
$$\alpha = 50.51°$$
$$\varphi = 39.50°$$
$$\Theta_2 = \alpha \mp \varphi = 50.51° \pm (39.50°) = -90° \; or \; 11.01°$$

The calculations yield the following

Table 5: Joint angles when robot at point 1 and 2

	Θ_1	Θ_2	Θ_3
Point 1	$-102.81°$	$65.28°$	$-88.78°$
Point 2	$-103.06°$	$67.62°$	$-88.18°$
Home	$0°$	$-90°$	$90°$

Dynamic Analysis

To perform the dynamic analysis and derive the Lagrange equations for the robot, the Euler-Lagrange Formulation below was used, focusing on the first two joints and links.

$$L = K - P$$

The translational and rotational kinetic energy terms for each link were calculated and the total kinetic energy was found using the equations described in the Methodology section. For a 2-DOF system (considering the first two joints and links):

$$q_1 = \theta_1$$
$$q_2 = \theta_2$$

Kinetic energy

For link 1 there is rotation about the z-axis yielding a velocity of the centre of mass:

$$\boldsymbol{v_1} = \begin{bmatrix} -\frac{l_1}{2}\sin(\theta_1) \\ \frac{l_1}{2}\cos(\theta_1) \\ 0 \end{bmatrix} \dot{\theta}_1$$

$$\boldsymbol{v_1^T} = \begin{bmatrix} -\frac{l_1}{2}\sin(\theta_1) & \frac{l_1}{2}\cos(\theta_1) & 0 \end{bmatrix} \dot{\theta}_1$$

Link 2 experiences planar motion, hence the velocity of the centre of mass is derived as:

$$\boldsymbol{v_2} = \begin{bmatrix} -l_1\sin(\theta_1) - \frac{l_2}{2}\sin(\theta_1+\theta_2) \\ l_1\cos(\theta_1) - \frac{l_2}{2}\cos(\theta_1+\theta_2) \\ 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

$$\boldsymbol{v_2^T} = \begin{bmatrix} -l_1\sin(\theta_1) - \frac{l_2}{2}\sin(\theta_1+\theta_2) & l_1\cos(\theta_1) - \frac{l_2}{2}\cos(\theta_1+\theta_2) & 0 \end{bmatrix} \begin{bmatrix} \dot{\theta}_1 \\ \dot{\theta}_2 \end{bmatrix}$$

Therefore,

$$\begin{aligned} K_1 &= K_{1:ROTATIONL} + K_{1:TRANSLATIONL} \\ &= \frac{1}{2}I_1\omega_1^2 + \frac{1}{2}m_1v_1^2 \\ &= \frac{1}{2}I_1\dot{\theta}_1^2 + \frac{1}{2}m_1\boldsymbol{v_1^T}\boldsymbol{v_1} \end{aligned}$$

$$= \frac{1}{2}I_1\dot{\theta}_1^2 + \frac{1}{2}m_1\left[\left(-\frac{l_1}{2}\sin(\theta_1)\right)^2 + \left(\frac{l_1}{2}\cos(\theta_1)\right)^2\right]$$
$$= \frac{1}{2}I_1\dot{\theta}_1^2 + \frac{1}{2}m_1\left(\frac{l_1^2}{4}\dot{\theta}_1^2\right)$$
$$= \left(\frac{1}{2}I_1 + \frac{m_1 l_1^2}{4}\right)\dot{\theta}_1^2$$

$$K_2 = K_{2:ROTATIONL} + K_{2:TRANSLATIONL}$$
$$= \frac{1}{2}I_2\omega_2^2 + \frac{1}{2}m_2 v_2^2$$
$$= \frac{1}{2}I_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2 + \frac{1}{2}m_2 \boldsymbol{v_2^T v_2}$$
$$= \frac{1}{2}I_2\left(\dot{\theta}_1 + \dot{\theta}_2\right)^2$$
$$+ \frac{1}{2}m_2\left[\left(-l_1 sin(\theta_1) - \frac{l_2}{2}sin(\theta_1+\theta_2)\right)^2\dot{\theta}_1^2 + \left(l_1\cos(\theta_1) + \frac{l_2}{2}\cos(\theta_1+\theta_2)\right)^2\dot{\theta}_1^2\right.$$
$$\left. + \left(-\frac{l_2}{2}\sin(\theta_1+\theta_2)\right)^2\dot{\theta}_2^2 + \left(\frac{l_2}{2}\cos(\theta_1+\theta_2)\right)^2\dot{\theta}_2^2\right]$$

$$K_{TOTAL} = \frac{1}{2}\sum_{i=1}^{2} K_{i:ROTATIONL} + \frac{1}{2}\sum_{i=1}^{2} K_{i:TRANSLATIONL}$$
$$K_{TOTAL} = \left(\frac{1}{2}I_1 + \frac{m_1 l_1^2}{4}\right)\dot{\theta}_1^2 + \frac{1}{2}I_2(\dot{\theta}_1 + \dot{\theta}_2)^2$$
$$+ \frac{1}{2}m_2\left[\left(-l_1 sin(\theta_1) - \frac{l_2}{2}sin(\theta_1+\theta_2)\right)^2\dot{\theta}_1^2\right.$$
$$+ \left(l_1\cos(\theta_1) + \frac{l_2}{2}\cos(\theta_1+\theta_2)\right)^2\dot{\theta}_1^2 + \left(-\frac{l_2}{2}\sin(\theta_1+\theta_2)\right)^2\dot{\theta}_2^2$$
$$\left. + \left(\frac{l_2}{2}\cos(\theta_1+\theta_2)\right)^2\dot{\theta}_2^2\right]$$

Potential Energy

$$P_1 = P_{1:GRAVITATIONAL}$$
$$= m_1 g\frac{l_1}{2}\cos\theta_1$$
$$P_2 = P_{2:GRAVITATIONAL}$$
$$= m_2 g\left[l_1\cos\theta_1 + \frac{l_2}{2}\cos(\theta_1+\theta_2)\right]$$
$$P_{TOTAL} = \sum_{i=1}^{2} P_{i:GRAVITATIONAL}$$
$$P_{TOTAL} = m_1 g\frac{l_1}{2}\cos\theta_1 + m_2 g\left[l_1\cos\theta_1 + \frac{l_2}{2}\cos(\theta_1+\theta_2)\right]$$

$$P_{TOTAL} = m_1 g \frac{l_1}{2}\cos\theta_1 + m_2 g l_1 \cos\theta_1 + m_2 g \frac{l_2}{2}\cos(\theta_1 + \theta_2)$$

The Lagrangian is given by

$$L = K - P$$

$$L = K_{TOTAL} = \left(\frac{1}{2}I_1 + \frac{m_1 l_1^2}{4}\right)\dot{\theta}_1^2 + \frac{1}{2}I_2(\dot{\theta}_1 + \dot{\theta}_2)^2$$
$$+ \frac{1}{2}m_2\left[\left(-l_1 sin(\theta_1) - \frac{l_2}{2}sin(\theta_1 + \theta_2)\right)^2 \dot{\theta}_1^2 + \left(l_1\cos(\theta_1) + \frac{l_2}{2}\cos(\theta_1 + \theta_2)\right)^2 \dot{\theta}_1^2\right.$$
$$\left. + \left(-\frac{l_2}{2}\sin(\theta_1 + \theta_2)\right)^2 \dot{\theta}_2^2 + \left(\frac{l_2}{2}\cos(\theta_1 + \theta_2)\right)^2 \dot{\theta}_2^2\right] - m_1 g \frac{l_1}{2}\cos\theta_1 - m_2 g l_1 \cos\theta_1$$
$$- m_2 g \frac{l_2}{2}\cos(\theta_1 + \theta_2)$$

Using the Lagrange equation:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{q}_i}\right) - \left(\frac{\partial L}{\partial q_i}\right) = \tau_i$$

For $q_1 = \theta_1$:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_1}\right) - \left(\frac{\partial L}{\partial \theta_1}\right) = \tau_1$$

For $q_2 = \theta_2$:

$$\frac{d}{dt}\left(\frac{\partial L}{\partial \dot{\theta}_2}\right) - \left(\frac{\partial L}{\partial \theta_2}\right) = \tau_2$$

KUKA Robot Language Program

As part of the analysis of the KUKA KR30HA, it was essential to develop a KUKA Robot Language (KRL) program to execute the series of movements. The objective was to ensure the robot could move safely and accurately from a home position, stop at two points, and then return to the home position. The code can be found in Appendix B.

To prioritise the safety and security of the robot's movements, the program includes a delay after each motion command. This ensures that the robot is securely in position before executing to the next movement.

The program was designed with an initialisation step inside the loop to set the tool and base data properly. This was executed each time the path was performed, ensuring consistent and accurate robot behaviour.

Conclusion

In conclusion, this report provided a detailed analysis of the kinematic and dynamic aspects of the KUKA KR30HA robot, demonstrating an understanding of robotic motion and control. The steps taken included determining the forward kinematics, inverse kinematics, and dynamic analysis using the Euler-Lagrange formulation.

The forward kinematics analysis enabled the determination of the end-effector's position and orientation based on joint parameters, using the Denavit-Hartenberg convention. The inverse kinematics analysis identified the joint angles required for the third link to reach specified target points. The dynamic analysis involved calculating the kinetic and potential energy of the robot's links 1 and 2, leading to the equations of motion.

Additionally, the development of a KUKA Robot Language program was written to demonstrate the practical application of the concepts discussed in this report. The program executed a defined motion path, while focusing on safety and accuracy.

The methodologies and findings presented in this report highlight the critical role of kinematic and dynamic analysis in enhancing the performance and reliability of industrial robots. This report serves as a guide for understanding and implementing robotic motion solutions for optimal robotic operation.

References

Utexas.edu. (2022). KUKA Programming KRL Examples - School of Architecture Digital Technologies Wiki - UT Austin Wikis. [online] Available at: https://wikis.utexas.edu/display/SOAdigitech/KUKA+Programming+KRL+Examples [Accessed 21 May 2024].

Robot Programming 1 KUKA System Software 8 Training Documentation. (n.d.). Available at: https://yazelin.github.io/ntcri/data/phase1/KUKA_Basic/doc/en/KUKA%20Robot%20Programming%201.pdf. [Accessed 21 May 2024].

Appendix

Appendix A: Technical Specifications of KR30HA Robot

Basic data

	KR 30 HA
Number of axes	6
Number of controlled axes	6
Volume of working envelope	27.24 m^3
Pose repeatability (ISO 9283)	± 0.05 mm
Weight	approx. 665 kg
Rated payload	30 kg
Maximum reach	2033 mm
Protection rating	IP64
Protection rating, in-line wrist	IP65
Sound level	< 75 dB (A)
Mounting position	Floor
Footprint	660 mm x 660 mm
Hole pattern: mounting surface for kinematic system	-
Permissible angle of inclination	-
Default color	Base frame: black (RAL 9005); Moving parts: orange (RAL 2003)
Controller	KR C4
Transformation name	KR C4: KR30HA_3 C4 FLR ZH02

Axis data

Motion range	
A1	±185 °
A2	-135 ° / 35 °
A3	-120 ° / 158 °
A4	±350 °
A5	±119 °
A6	±350 °
Speed with rated payload	
A1	140 °/s
A2	126 °/s
A3	140 °/s
A4	260 °/s
A5	245 °/s
A6	322 °/s

Mastering position	
A1	0 °
A2	-90 °
A3	90 °
A4	0 °
A5	0 °
A6	0 °

Appendix B: KUKA Robot Language Program

```
DEF TwoPointMotionPath()

    ; Initialization code to set tool and base data
    BAS(#INITMOV, 0)

    ; Define origin of tool
    $TOOL = {X 0, Y 0, Z 0, A 0, B 0, C 0}

    ; Define origin of base
    $BASE = {X 0, Y 0, Z 0, A 0, B 0, C 0}

    ; Define points
    DECL Point1 = {X 91, Y - 1139, Z 1392, A 0, B 0, C 0}
    DECL Point2 = {X 91, Y - 1117, Z 1447, A 0, B 0, C 0}
    DECL MasteringPos = {X 0, Y 0, Z 0, A 0, B 0, C 0}

    ; Call commands followed by a delay to ensure the robot is safely in position

    ; Move to mastering position
    PTP Mastering
    WAIT SEC 2

    ; Move to Point 1
    PTP Point1
    WAIT SEC 2

    ; Move to Point 2
    PTP Point2
    WAIT SEC 2

    ; Return to mastering position
    PTP MasteringPos
    WAIT SEC 2
END
```

Addendum

Marking Rubric

<u>ASSESSMENT (2024)</u>

Graduate Attribute 2: Application of scientific and engineering knowledge

Module: Advanced Manufacturing Systems 4

(EAMV401)

Learning Outcome: Given a robot motion problem, demonstrate competence to apply mathematical and physical science knowledge to solve the dynamic and kinematic problem of the system using Excel.

Candidate Name: Kyle Flanegan **Student No: s220324085**

Exit Level Outcome	Associated Assessment Criteria	Performance (Assessment Rubric)				
		Unsatisfactory <45	Marginally Unsatisfactory 45-49	Satisfactory 50-59	Exceeds Requirements 60-74	Excellent 75-100
GA 2	The candidate applied the skills in **a Class assignment**	WEAK or NO	LIMITED	STANDARD AND SATISFACTORY	DETAILED	INNOVATIVE AND THOROUGH
	1. Application of the systematic theory approach to Robotic Kinematics and Dynamics aspects of a complex 6-axis manipulator system. (Ch 2,3 & 7)	No or weak indication of an approach to resolve the Kinematic and dynamic solutions.		Satisfactory implementation of the theory of Kinematics and Dynamics to determine the equations of motion for the robot.		Thorough implementation of all aspects of the theory of robotic kinematics and dynamics to reach the equation of motion for the robot.
	2. Describe and discuss the derivation of the Dynamics and Kinematics in engineering terms to show Engineering specialist knowledge.	No or insignificant discussion of the Kinematic or Dynamic derivation.		Satisfactory discussion of the Kinematic and Dynamic solution to the Robot manipulation problem.		Exceptional insight shown during the discussion of the Kinematic and Dynamic solution to the Robot manipulation problem.
	3. Utilise advanced mathematical analysis tools to generate the Kinematic and Dynamic solution. (Ch 2,3 & 7)	No or little utilisation of advanced mathematical analysis tools to reach the Kinematic and Dynamic solution. No Kinematic and Dynamic solution indicated		Satisfactory and correct application of advanced mathematical analysis tools to reach the Kinematic and Dynamic solutions. Kinematic and Dynamic solution indicated		Extensive and possibly new mathematical analysis not covered in the module used to reach desired Kinematic and Dynamic solution indicated.
	4. Utilise the appropriate software to calculate the solution to the	No or weak implementation of Dynamic and Kinematic solution in Excel.		Satisfactory (Enough/Required) implementation of Dynamic and		Extensive implementation of Dynamic and Kinematic solution in

	derived dynamic characteristics for a proposed manipulator path. (Ch 2,3 & 7)			Kinematic solution in Excel. Satisfactory use of Excel functions.		Excel. Exceptional use of the software to show the solution.
	5. Self-study component: Generation of KUKA robot programme to perform the motion path.	No or incorrect KUKA software programme generated.		Satisfactory robot programme reported. Understanding of the various functions indicated.		Thorough robot programme reported which may be used on industrial robot with conceptual understanding of the programming evident.
Tally						

Has GA been satisfied? Yes or No.	DC